MSL Technical Guide 27

Impedance (RLC) Standards

Introduction

This guide is about the ac measurement of resistors, inductors and capacitors (R, L and C). In particular it is aimed at helping owners of impedance standards make the best use of their equipment, starting in Part I with understanding the information provided in a typical calibration certificate. Discussion is limited to standards that might be measured at frequencies anywhere from 20 Hz to 1 MHz. While resistance standards are also commonly measured all the way down in frequency to dc, inductors and capacitors are rarely measured below 50 Hz.

Modern impedance measuring instruments generally perform very well, so long as the guidance offered in the user manual is followed. More likely sources of confusion come from relating what has been measured by an instrument to exactly what really needs to be known about an electrical circuit. Advice on ensuring that typical laboratory standards are measured correctly is given in Part II.

Part I: Understanding the Certificate

Terminology

In common usage the term impedance is often used to cover all RLC measurements, but the term 'immittance' more correctly refers to both impedance and admittance. Immittance calibration certificates should be clear and unambiguous. However, they do rely on using some technical terms that have a special meaning in the context of the calibration. In particular the terms two-terminal, three-terminal, two-terminal-pair and four-terminal-pair are more than just casual descriptions of the terminals on a standard. They refer to specific connection and defining conditions for immittance measurement.

References in the certificate to making connections to 'screen', 'ground' or 'low potential' should not be treated as synonymous. Rather, these terms carefully reflect the labelled terminals on the standards so that you can reproduce a properly defined connection as used during the calibration.

Also, note that the use of the suffixes '–ance' and '–or' is quite deliberate with, for example, the term 'inductor' being used to describe the actual device while 'inductance' is used to describe the electrical property. A full electrical description of an inductor would include its properties of resistance and capacitance as well as its inductance.

Definitions

For the simple circuit in Figure 1 the impedance, Z, and admittance, Y, are given by

$$Z = \frac{V}{I},\ Y = \frac{I}{V}$$

where V is the applied voltage and I is the resultant current flowing through the immittance.

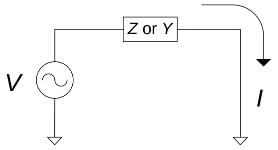
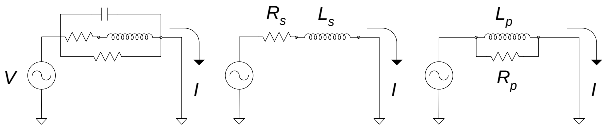


Figure 1. Immittance measurement.

The value of an immittance is reported in a certificate as the values of two components of an equivalent network consisting of a resistive and a reactive component. These components may either be in parallel or series connection. The two-component equivalent network only has the same immittance as the standard at the measurement frequency.



(a) (b) (c)

Figure 2. Equivalent circuit of an inductor (a), reported series equivalent circuit (b) and reported parallel equivalent circuit (c).

For example an inductor might be well modelled by a network such as in Figure 2 (a). The resistance of the windings is modelled as a resistance in series with the inductance of the windings; the inter-winding capacitance is modelled as a single capacitance in parallel with the windings and the loss in that capacitance is modelled as a resistance in parallel with the windings. When measured however, the value will be reported simply as either the series or parallel equivalent two-component networks as in Figure 2 (b) and (c). All three networks have the same impedance at the given frequency with

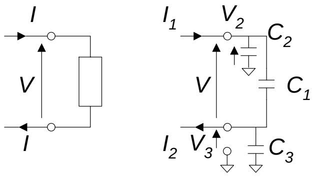
$$R_s = \frac{R_p}{1 + \left(\frac{R_p}{\omega L_p}\right)^2},\ L_s = \frac{L_p}{1 + \left(\frac{\omega L_p}{R_p}\right)^2}$$

where $\omega = 2\pi f$ and f is the frequency. These calibration values do not give the values of the network components in Figure 2 (a). In fact measurements at more than one frequency would be required to determine what sort of lumped-component network best described a particular inductor.

It is generally the case that an immittance standard will be close to being a pure inductor, capacitor or resistor. The unwanted, for example, conductance of a capacitor or inductance of a resistor is referred to as a residual component. Sometimes the residual component values might be reported as a loss angle, time constant, dissipation or quality factor. These alternative representations are simply calculated from the two-component series or parallel network values and provide no additional information about the standard. However, some impedance meters may provide more significant digits for an alternative representation of the residual component. As always, care is needed to ensure that these additional digits are genuinely meaningful.

Connections and Defining Conditions

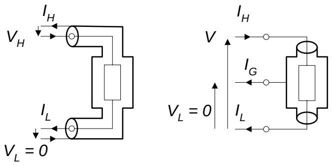
Several ways have been devised for making connections to standard impedances so that they can be measured reproducibly. An excellent description of all the approaches can be found in [1]. The choice of connection method depends on the manufacturer's design of the standard and the available impedance meter. For each physical connection method there is also a set of electrical defining conditions that must be met at the connections. If it is not possible to reproduce the connection method used in the calibration certificate, an estimate of the likely difference due to using an alternative connection must be made.



(a) (b)

Figure 3. Two-terminal immittance (a) and two-terminal capacitor with stray capacitance (b).

In the two-terminal connection in Figure 3(a) the impedance is given by $Z_\mathrm{a} = V/I$, where it is assumed that all the current entering the upper terminal exits from the lower terminal. This connection is useful for measuring unpackaged components. However, for ac impedances the current can also flow through, for example, stray capacitances to surrounding objects. This is shown in Figure 3(b) where some of the current entering the upper terminal, I_1, will flow through C_2 to a common point if V_2 is not zero. There will also be current through C_3 if V_3 is not zero. Suppose C_2 and C_3 are only at the picofarad level and C_1 is several microfarads, then the current loss is negligible compared to the current through C_1. However, if C_1 is itself only a few picofarads, the current loss will give a large error.



(a) (b)

Figure 4. Two-terminal-pair (a) and three-terminal (b) connections to an impedance standard.

For higher impedance standards (e.g. low value capacitors) the two-terminal-pair connection of Figure 4(a) is preferred. Note that 'terminal-pair' refers to a coaxial connection, one terminal for the inner and one for the screen. With a permanent screen around the impedance element, any stray capacitances are now a permanent part of the standard. In terms of Figure 3(b), C_2 and C_3 are now fixed. By now an adding the defining condition that voltage difference between inner and outer at the lower port must be zero ($V_L = 0$) the difference between I_H and I_L is fixed. The impedance for the two-terminal-pair connection is, therefore, given by $Z_\mathrm{b} = V_H/I_L$. With this connection and defining condition, the value of even a 10 pF capacitor is unaffected at, say, 1 kHz when using connecting leads of several hundred picofarads.

Many older impedance standards were designed to be measured as three-terminal impedances as shown in Figure 4(b). The three terminals are likely to be banana or screw terminals rather than coaxial. This connection can be thought of as an approximation to a two-terminal-pair version. If the potential between the screen and the lower terminal is maintained at zero ($V_L = 0$) then the impedance is given by $Z_\mathrm{c} = V_H/I_L$.

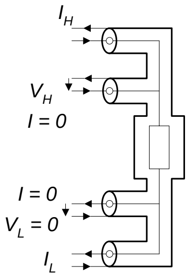


Figure 5. Four-terminal-pair impedance. 'H' and 'L' refer to high and low potential with respect to the coaxial screen.

For lower impedance standards (e.g. low value inductors and resistors) the series impedance (R and L) of the coaxial leads directly affects the two-terminal-pair value. Lead impedance adds in series with the impedance of the standard. The four-terminal-pair connection of Figure 5 removes the effect of both the capacitance and series impedance of long coaxial leads. This is conditional on the measurement circuit now satisfying two defining conditions.

Firstly it is required that there is zero current flowing in the high potential coaxial connector. In effect V_H is measured with an infinite input impedance vector voltmeter. Secondly it is required that the there is no potential difference between the inner and the outer of the low potential coaxial connector. This is the same as for the two-terminal-pair definition, ensuring that there is negligible current flowing through the stray capacitance to shield at the low potential terminal. The four-terminal-pair impedance is then given by $Z = V_H/I_L$.

Note that for two-terminal-pair measurements at higher frequencies, the error due to cable capacitance and inductance increases rapidly with frequency. Therefore the four-terminal-pair definition is preferred for all standards when operated at higher frequencies.

Part II: Using the Standards

Capacitors

On receiving your calibrated capacitance standard with a fresh calibration certificate, one of the first tasks is to measure the standard with your own equipment in order to check that there has been no significant change in the standard while it was away for calibration.

In order to use the new certificate value to calibrate your measurement system you need to be sure that you are able to reproduce the measurement conditions as described in the certificate. If the temperature is different you will need to apply a correction. For a solid dielectric capacitor the temperature coefficient could be as high as 0.02 %/°C.

The certificate also describes the connection used for measurement and this needs to be duplicated with your own equipment. The simplest situation is for a four-terminal-pair standard that has been calibrated as such, and where your impedance meter is also four-terminal-pair. If the capacitance standard has exposed terminals, such as in Figure 6, then particular care is needed to minimise the capacitance due to connecting leads.

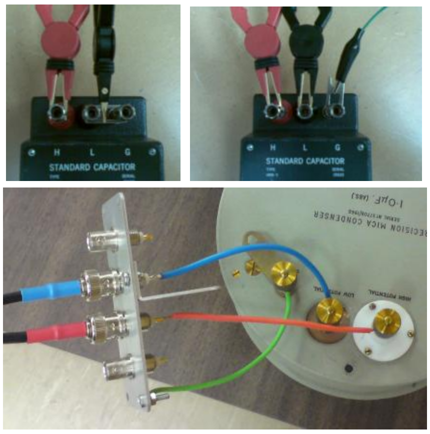


Figure 6. Capacitors connected as (clockwise from top left), two-terminal, three-terminal and two-terminal-pair impedances.

The upper images in Figure 6 show screened Kelvin-clip leads that are connected to a four-terminal-pair impedance meter. These leads have a stray capacitance of less than 0.1 pF for typical terminal spacings. This stray capacitance can easily investigated by varying the distance between the clips with the standard disconnected. While the 0.1 pF presents a negligible error for a 1 µF capacitor, it is a 1% error for a 10 pF standard. Note that some commercially available instruments are capable of resolving better than 0.0001% of a two-terminal-pair 10 pF standard.

It is common for a capacitance box calibration certificate to give the change in capacitance going from a dial setting of zero to the indicated dial setting. On the assumption that the connection leads remain undisturbed during the dial change, it is possible to provide a much lower uncertainty in capacitance change than for the total capacitance. The certificate will also quote the capacitance measured when all dials are set to zero and this ‘zero’ capacitance will have an uncertainty that takes into account the issue of stray capacitance between the leads.

The lower image in Figure 6 shows an adaptor plate for using a two-terminal-pair capacitance bridge to measure a three-terminal capacitor. The unscreened leads have some stray capacitance, but again this is negligible for a 1 µF capacitor. For higher value capacitors it is more important to control the resistance of the leads and contact with the terminals. A relatively small series resistance can have a significant effect on the measured loss angle (dissipation factor or parallel conductance). The phase angle, *δ*, is given by

$$\delta = \tan^{-1}(\omega R_S C_S)$$

where $\omega = 2\pi f$ and f is the frequency, and R_s and C_s are the series resistance and capacitance respectively.

For a loss-free 1 µF capacitor measured at 1.6 kHz, a series lead or contact resistance as low as10 mΩ results in a measured loss angle of 100 µrad. This is significant for a solid dielectric 1 µF capacitor that might have a loss angle, due to its intrinsic material properties as low as 50 µrad.

A warning about connectors on some two-terminal-pair standards (two coaxial connectors) is that they might have the shell of one of the coaxial connectors electrically insulated from the case. This makes the default connection three-terminal rather than two-terminal-pair. You should check the change seen by your impedance meter caused by clipping a lead from the outer of the insulated coaxial panel connector to the case of the standard. If this change is significant it might be worthwhile to modify the standard to a true two-terminal-pair configuration.

Measurement frequency is an important parameter. Manufacturer’s data should be used to determine the likely frequency coefficient at the calibration frequency. A mix of connector inductance, eddy currents and dielectric properties contribute to the overall frequency dependence. Also note that the effect of connecting leads generally increases as the square of frequency. For example, using a few metres of coaxial cable to measure a two-terminal-pair capacitance will cause an error of less than 0.00001% at 1.0 kHz, but the same cables at 1 MHz would give a 10% error. A four-terminal-pair connection is designed to eliminate this error.

Finally the voltage at which a capacitor is measured might affect the value. This should not be significant when standards are measured with a typical impedance/capacitance meter operating at a few volts.

Inductors

While the electric fields of a capacitor can be fully contained within the screening case of a capacitor, some small part of the magnetic field of an inductor will always extend beyond the case of the inductor. A standard inductor is designed to minimise its external magnetic field. However, it is important to ensure that all magnetic and conducting material is kept sufficiently far away from the inductor while it is being measured. A practical test is to take, for example, a small steel hand tool (check that it sticks to a magnet) and look at the change in impedance meter reading as the tool is brought up to the standard. Depending on the required uncertainty, a 0.5 metre radius exclusion zone should be adequate. Be particularly alert to things like steel framing under an insulating bench surface.

Unlike most standard capacitors, all standard inductors have a significant resistance component. The phase angle of an inductor, *δ*, is given by

$$\delta = \tan^{-1}\left(\frac{R_s}{\omega L_s}\right)$$

where $\omega = 2\pi f$ and f is the frequency, and R_s and L_s are the series resistance and inductance respectively of the two-component series-equivalent network.

For a high-accuracy 1 mH standard inductor with a 0.8 Ω series resistance, the resulting loss angle is 0.08 rad, several orders of magnitude greater than for the best capacitance standards. As copper wire is usually used for winding the inductor, the resistance will have a temperature coefficient of about +0.4 %/°C. This is likely to be two orders of magnitude greater than the +0.003 %/°C temperature coefficient of inductance for the same inductor. This difference in sensitivity can be used to advantage, as the resistance of the inductor can be the best indicator of its temperature. If the resistance can be measured to an accuracy of 0.2%, using a four-wire dc resistance meter, then it is a good alternative to using a separate nearby thermometer.

When making connections to a standard inductor, care is needed to ensure that there is minimal coupling between any magnetic field from the inductor and the screens of the coaxial cables from the impedance meter. This can be achieved by either gently twisting the coaxial cables together or at least strapping them into a single bundle. Again, the practical test is to see by how much the impedance meter reading changes when the cables are brought together.

Most inductance standards are manufactured as three-terminal devices. The most reproducible measurements of these are achieved by measuring the change in impedance from with to without a shorting link in place as shown in Figure 7. It is important to ensure the cables remain in the same position throughout the measurements, so that the change is only due to the changing link position and not any magnetic coupling effects with the leads.

Figure 7. Three-terminal inductor connection. On left with zero link in place and on right with the link disconnected.

Measurement frequency is again an important parameter. While the two-element equivalent network of an inductor does not explicitly include a capacitor, in fact there is always capacitance associated with a wound inductor. A simple model network with just a capacitor in parallel with an inductor has an inductance that varies with frequency as

$$L(\omega) = \frac{L(0)}{(1-\omega^2 L(0)C)}, \quad f_0 = \frac{1}{2\pi\sqrt{L(0)C}}$$

where $\omega = 2\pi f$ and f is the frequency, and f_0 is the self-resonance frequency.

The self-resonance frequency of a standard inductor can be quite low, for example 15 kHz for a 1 H inductor; essentially the inductance is in parallel with a 120 pF capacitance. For this 1 H inductor its inductance changes 1% going from dc to 1.6 kHz and the rate of change of inductance with frequency continues to increase up to self-resonance. Resonant effects can also be observed in capacitors where the capacitance interacts with the small inductance of the internal connections.

At higher frequencies it can be useful to simulate an inductor using a simple RC network. The two networks in Figure 8 are equivalent as seen from the four terminals. When measured as a two-terminal-pair component the measured impedance is Z, where

$$Z = R + j\omega C\frac{R^2}{4}.$$

Remember that meeting the defining conditions of the two-terminal-pair connection means that only Z is measured. The other two impedances have no effect.

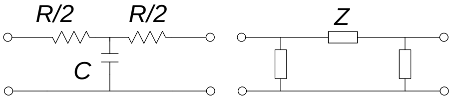


Figure 8. T-network on left with equivalent pi-network on the right.

For example, using two 1 kΩ resistors and 100 nF capacitor the measured impedance is 100 mH in series with a 2 kΩ resistor. At 1 MHz this is an inductor with a phase angle of 3 mrad. The phase angle increases as the frequency decreases. This approach takes advantage of the high stability of available capacitors and resistors and benefits from avoiding the stray magnetic field coupling issues common for wound inductors.

Resistors

The most long-term-stable resistance standards are usually maintained at dc rather than ac. For modern precision bulk foil resistors there is very little frequency dependence of resistance, and the ac resistance at 1.6 kHz is likely to be within 0.0001% of the dc value.

Depending on your dc resistance measurement capability, it is good practice to cross-check the dc and ac values of your ac resistance standards. When doing so, it is important to realise that the four-terminal-pair connection of Figure 4 effectively connects the impedance of the screen in series with the impedance of the internal element. The screen resistance that is added is the dc four-wire dc resistance that is measured by connecting to the outer shells of the four coaxial connectors on the standard (with all impedance meter cables disconnected). When the four coaxial connectors are located close together on the outer case you would expect a negligible resistance. If the connectors are widely spaced, several milliohms of resistance might be added.

Resistance thermometers are often measured at relatively low frequencies (< 100 Hz) using a simpler coaxial four-terminal connection [2], that is adequate below 1 kΩ and when knowing the phase angle is unimportant. In general, however, four-terminal-pair connection is strongly recommended for measuring ac resistors.

With the four-terminal-pair definition there is no strict need to measure an initial zero impedance. However, a zero impedance standard is useful for calibrating an impedance meter.

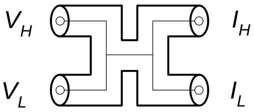


Figure 9. Four-terminal-pair zero impedance.

The coaxial circuit in Figure 9 is a perfect four-terminal-pair zero impedance that can be constructed from coaxial goal-post/tee and barrel connectors. It is important to note that the two potential (V_H and V_L) and two current (I_H and I_L) connectors are each paired in their respective tees before the potential and current pairs are connected with the barrel. The alternative connection would see the impedance of the barrel connector being measured rather than zero impedance.

Impedance Meters

There is a long history of measurement bridges for various types of electrical impedances [3], with many of these bridges predating the development of coaxial two- and four-terminal-pair impedance measurement systems. If you are using an older bridge system it will still be possible to make good use of calibrated standard impedances, but you should approach MSL for advice on tests that will establish any additional uncertainty components for your bridge.

A modern laboratory impedance meter will most likely be configured as either a two- or four-terminal-pair system with BNC panel connectors. There are some simple checks to ensure that best use is made of the calibrated impedance standard for calibrating the impedance meter.

- Check the meter range and preferably manually lock the meter to the most appropriate (highest resolution) range when carrying out the calibration.
- Similarly choose the display option (R_s, L_s; G_p, C_p etc.) that offers the maximum resolution to avoid display resolution being a dominant uncertainty term.
- Check the zero for the range being used against a properly constructed coaxial zero impedance (short circuit) or zero admittance (open circuit). Make sure that the screens of the leads are connected when measuring the open circuit.
- Check the frequency and amplitude of the signal being applied (note that due to finite output impedance the voltage at the standard can be significantly lower than the selected voltage).
- Check the change caused by doubling the length of the coaxial cables connecting to the standard.

The last check is to ensure that the meter is adequately meeting the necessary defining conditions. For a two-terminal-pair meter the effect of the additional lead length will depend on the impedance it is connected to. A four-terminal-pair meter should show no significant change. In practice all meters have their limitations, but these should be clear from the manufacturer's specifications.

Conclusion

A calibrated impedance meter is a good starting point for making reliable impedance measurements. When making measurements of circuit components and particularly impedance sensors, you need to decide on the most appropriate connection method to use. This guide provides a basis for making those decisions, but further reading of [1] is strongly recommended for more challenging applications.

References

[1] Shakil Awan, Bryan Kibble and Jürgen Schurr, *Coaxial Electrical Circuits for Interference-Free Measurements*, Institution of Engineering and Technology, London, 2011.

[2] MSL Technical Guide 18: *Resistance measurement for thermometry*, http://msl.irl.cri.nz.

[3] Hague B (revised by T R Foord), *Alternating current bridge methods, 6th edition*, Pitman, London, 1976.

Prepared by Keith Jones, November 2011.