MSL Technical Guide 24

Dew-Frost Error and the Chilled-Mirror Hygrometer

Summary

This technical guide explains how to account for, and partially correct, the error in a chilled-mirror hygrometer measurement when it is not known whether there is super-cooled dew or frost condensed on the mirror. It starts with a short discussion of how a chilled mirror hygrometer works, how dew-point temperature and vapour pressure can be determined, and the cause of dew-frost ambiguity. The following sections show how to estimate the values and uncertainties of vapour pressure, dew point and relative humidity when it is not known whether the condensate is dew or frost.

Introduction

A chilled-mirror hygrometer is used to determine the dew-point or frost-point temperature or the relative humidity of a humidified air-stream. The hygrometer measures the dew point by cooling a tiny metal mirror down until water vapour condenses on the surface. When the air is sufficiently dry, the water vapour will condense on the chilled-mirror as frost, otherwise the condensate will be in liquid form (dew), even though the mirror temperature may be below the ice-point at 0 °C.

The condensed water (condensate) is detected optically, usually by monitoring the decrease in specular reflection (or the increase in scattered light) from a light source. The mirror temperature is then controlled so that there is no increase or decrease in the light reflected, indicating that the thickness and structure of the condensate layer is unchanging and, therefore, that the vapour pressures of the condensate and of the air-stream are in equilibrium. The mirror temperature, measured using an embedded resistance thermometer, thus indicates the dew-point (or frost-point) temperature of the humidified air stream.

Vapour pressure over water or ice at 0 °C is about 611 Pa. When the vapour pressure is greater than this, the condensate phase will always be dew. When the vapour pressure is less than 611 Pa, the condensate can be dew or frost. Liquid water at temperatures below 0 °C is said to be super-cooled, and may continue in this semi-stable (metastable) state for some time, and has been observed for periods exceeding 6 hours at temperatures above –24 °C and for shorter periods at lower temperatures. On the other hand, the super-cooled dew-droplets may at any time transform to the lower energy phase, ice, but not vice versa. Therefore, a chilled mirror hygrometer reading less than 0 °C, may indicate the presence of frost, of super-cooled dew or of some transitional state in which both phases are present. But while a vial of supercooled water might freeze nearly instantly, a layer of fine dew-droplets may take some time.

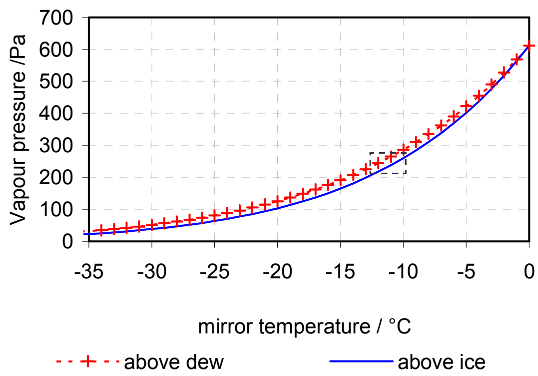

Figure 1. Vapour pressure above super-cooled dew and ice. Note that for the same vapour pressure, the super-cooled dew-point temperature is less than the frost-point temperature. The detail within the dashed-line box is shown in Figure 2.

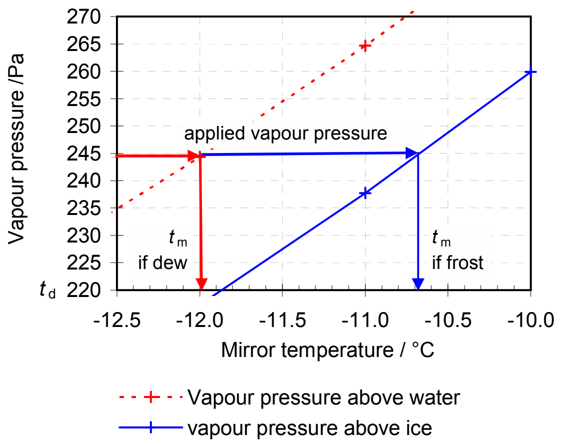

Figure 2. The same vapour pressure could result in a range of mirror temperatures t_m depending on whether there is super-cooled dew or frost on the mirror, or some transitional state as dew droplets freeze.

Figures 1 and 2 show the vapour pressure above dew and ice. Note that for the same vapour pressure, the super-cooled dew-point temperature (dew point) is less than the frost-point temperature (frost point). If the vapour pressure is 244 Pa, a mirror with super-cooled water will have a temperature of about –12 °C, a mirror with a frost layer will be at a temperature of –10.7 °C (see Figure 2). A mirror in which the dew-layer is turning to frost will be at some intermediate temperature.

Dew-frost ambiguity can contribute considerable uncertainty to humidity measurement using a chilled-mirror hygrometer. To address the issue some chilled-mirror hygrometers allow visual inspection of the mirror using some form of microscope. However, positive identification of dew or frost is not always possible. Another option is to temporarily lower the mirror temperature sufficiently to force frost. Most chilled mirror hygrometers provide means for the user to do this manually, but care is required to ensure that the frost layer is not melted once the hygrometer's normal control mode is regained, and the mirror temperature is allowed to swing above 0 °C. Fortunately, modern "state-of-art" hygrometers have been designed to "force frost" and then keep the mirror temperature below zero while finding the new balance point.

The visual inspection and automatic options can be expensive, and for many industrial applications, manual forcing of frost is not practical. Consequently, there is often a need to manage the potential error and account for error in an uncertainty budget. To this end, in the rest of this guide, we assess the possible errors in vapour pressure, in dew point and frost point, and in relative humidity, when the condensate phase is not known, and show how to apply corrections to reduce the uncertainty.

Correcting Vapour Pressure Error

If the condensate phase on the mirror is known, the vapour pressure can be determined from the mirror temperature t_m using a graph as in Figure 1, or, more accurately, using reference equations which are readily available (e.g. [1,2]).

If the condensate phase is unknown, the *most probable values* of vapour pressure are those corresponding to frost or dew at t_m, although the actual vapour pressure could be anywhere between the two. The maximum error in calculated vapour pressure Δp occurs when a false assumption about the condensate is made – the condensate is dew but is assumed to be ice or vice versa (Figures 3 and 4). Thus the maximum error is

$$\Delta p = p(t_m)_d - p(t_m)_f$$

where $p(t_m)_d$ and $p(t_m)_f$ are the vapour pressures at temperature t_m calculated over dew and ice, respectively. For example, in Figure 3 the mirror temperature of –12 °C indicates vapour pressures between 217 Pa and 244 Pa, so that $\Delta p = 27$ Pa.

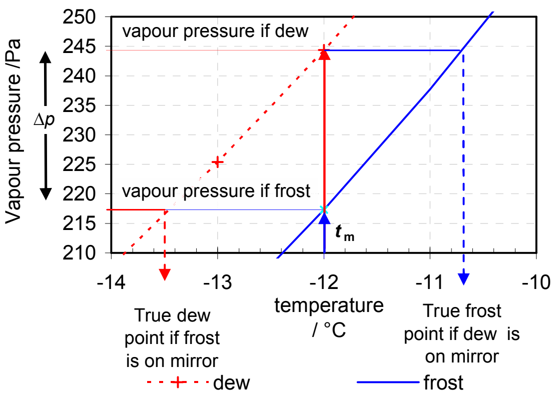

Figure 3. If the condensate phase is unknown, a range of vapour pressures is possible, depending on whether there is actually dew or frost, or both, on the mirror. Falsely assuming dew or frost leads to a maximum error of magnitude Δp.

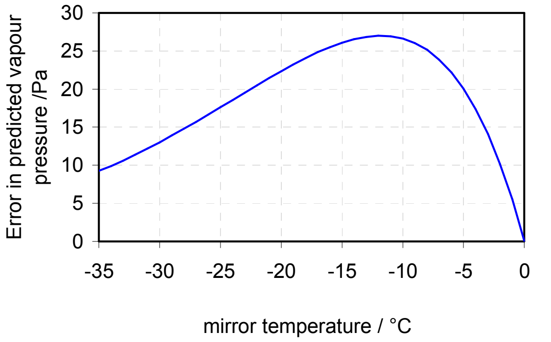

Figure 4. Maximum error in predicted vapour pressure for a false assumption of the condensate phase. This is just the difference Δp between the curves in Figures 1 to 3.

However, rather than making a potentially false assumption of frost or dew (thus, maximising the potential error), we may halve the potential error by using the average value of the vapour pressure p_m; that is,

$$p_m = \frac{p(t_m)_d + p(t_m)_f}{2} . \quad (1)$$

In the absence of information about the mirror state, we can call p_m the corrected value of vapour pressure. We next need to calculate the uncertainty associated with error in p_m.

If we consider any particular measurement as a random sample from a uniform distribution of width equal to the possible range in vapour pressure, the standard uncertainty in p_m is given in terms of the maximum error Δp as

$$u(p_m) = \frac{\Delta p}{2\sqrt{3}} .$$

Figure 5 shows the *relative error*, i.e., the error expressed as a percentage of the vapour pressure calculated over ice (dotted line) or over dew (dashed line), or as a percentage of the average (corrected) value p_m (thick solid line). From this graph, we can see that the relative error $\Delta p / p_m$ is approximately a linear function of t_m, i.e.,

$$\frac{\Delta p}{p_m} \approx -t_m \% .$$

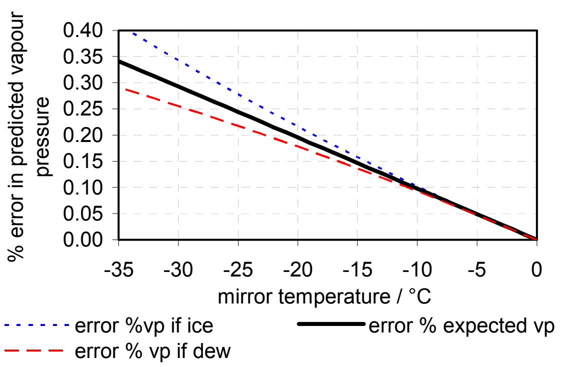

Figure 5. Error as a percentage of the actual vapour pressure if condensate is frost or dew (dotted and dashed lines) and error as a percentage of the "expected value" of vapour pressure (thick solid line, see text for details).

Consequently, we can write the relative uncertainty in the expected value of vapour pressure as

$$u_{\mathrm{r}}\left(p_{\mathrm{m}}\right) \approx -\frac{t_{\mathrm{m}}}{2\sqrt{3}}\% \approx -0.3 \cdot t_{\mathrm{m}}\%. \qquad (2)$$

In our example above, when $t_m = -12$ °C, $p_m = 230.8$ Pa and $u(p_m) = 8.3$ Pa. If we use twice the standard uncertainty as the expanded uncertainty at the 95% level of confidence, we see that the resulting confidence interval of 214.2 Pa ≤ p_m ≤ 247.4 Pa just encloses the vapour pressures corresponding to frost and dew on the mirror (217 Pa and 244 Pa, respectively).

Correcting Dew and Frost Point Error

Usually it is convenient to estimate the dew point or frost point when the condensate phase is unknown. The error in either dew point or frost point following a false assumption of the condensate phase is presented in Figure 6.

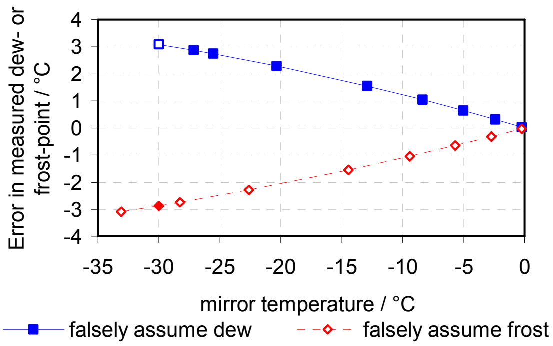

Figure 6. Error in dew point and frost point resulting from a false assumption of the phase of condensate on a chilled-mirror (see text for details).

If, for example, the chilled-mirror hygrometer reading is –30 °C, a *false* assumption that there is frost on the mirror (red diamonds) will give a –2.9 °C error in frost point, since the true frost point would be –27.1 °C. A false assumption of dew on the mirror (filled blue squares) will give a +3.1 °C error in dew point, since the true dew point would be 33.1 °C. The error is approximately –0.1 times the falsely assumed dew point or +0.1 times the falsely assumed frost point.

To minimise the potential error, we use the corrected vapour pressure p_m (equation (1)) and the relevant vapour pressure equations to calculate the *corrected* dew point t_d and frost point t_f and their uncertainties. It turns out that over a useful range from 0 °C to –30 °C, the corrected dew point t_d and frost point t_f may be approximated by

$$t_{\mathrm{d}} \approx 1.05 \cdot t_{\mathrm{m}} \text{ and } t_{\mathrm{f}} \approx 0.95 \cdot t_{\mathrm{m}}, \qquad (3)$$

with standard uncertainties given by

$$u\left(t_{\mathrm{d}}\right) \approx -0.034 \cdot t_{\mathrm{d}} \text{ and } u\left(t_{\mathrm{f}}\right) \approx -0.034 \cdot t_{\mathrm{f}}, \qquad (4)$$

respectively.

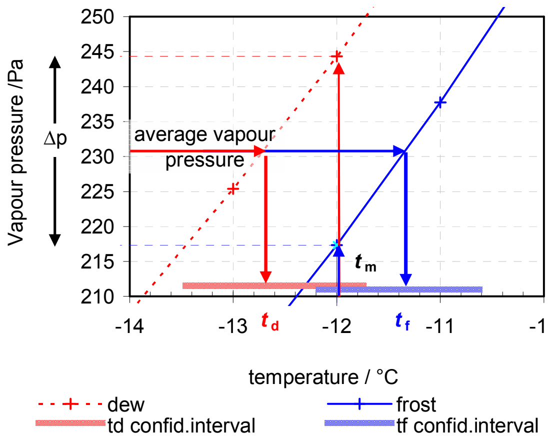

Figure 7. Determination of the corrected dew point or frost point and associated confidence intervals from the mirror temperature t_m when the condensate phase is unknown (see test for details).

For example, as is illustrated in Figure 7, when $t_m = -12$ °C, the corrected dew point is $t_d = -12.6$ °C ± 0.9 °C and the corrected frost point is $t_f = -11.4$ °C ± 0.8 °C. Here, again, the expanded uncertainties are calculated for a 95% level of confidence using twice the standard uncertainties.

Correcting Relative Humidity Error

Relative humidity h at dry-bulb temperature t is defined as the ratio of the actual water vapour pressure p (calculated from the measured dew or frost point) to the saturation vapour pressure p_s; that is, the vapour pressure that would be measured if the gas was saturated at t. Thus, as a percentage

$$h \approx \frac{p}{p_{\mathrm{s}}} \cdot 100\ \%\mathrm{rh}, \qquad (5)$$

where we have ignored the enhancement factor since the error in doing so is insignificant compared to the error deriving from dew-frost ambiguity. Dew-frost ambiguity will only be an issue when the vapour pressure is less than 611 Pa ($t_m < 0$ °C), so the maximum relative humidity for which dew-frost ambiguity is an issue depends on the dry-bulb temperature. When $t = 20$ °C, $p_s \approx 2339$ Pa, so dew-frost error is possible when $h < 611/2339$, i.e. $h < 26$ %rh. Similarly, at 10 °C, $p_s \approx 1230$ Pa, so to be affected by dew-frost error, h must be less than 611/1230, i.e. $h < 50$ %rh.

In Figure 8, relativity humidity is plotted as a function of dew point and dry-bulb temperature, and in Figure 9 the error in relative humidity following a false assumption of the condensate phase is presented for the same range of dry-bulb temperature.

The shape of the error curves is due to two competing effects. The magnitude of the dew point error increases with decreasing dew point and, hence, with decreasing h. At the same time the sensitivity of the relative humidity to dew point decreases. The maximum error occurs at relative humidities corresponding to a dew point of –12.5 °C (i.e., $p \approx 234$ Pa).

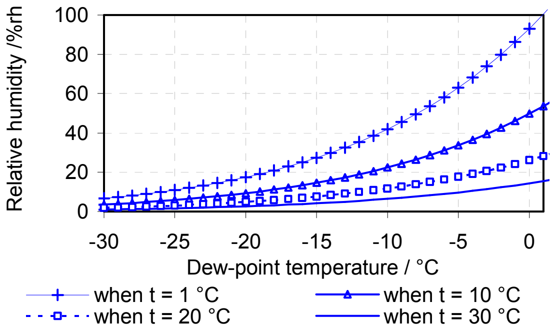

Figure 8. Relative humidity as a function of dry-bulb temperature *t*, for dew point temperatures less than 0 °C.

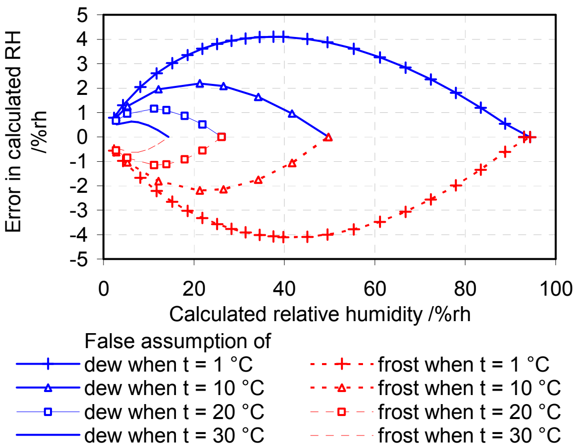

Figure 9. The error in calculated relative humidity based on a false assumption of dew or frost on a chilled mirror.

To minimise the potential error in relative humidity when the condensate phase is unknown, calculate the *corrected* dew point or frost point using equation (3), then calculate the *expected* vapour pressure and substitute this into equation (5) to find the expected relative humidity *h*. The standard uncertainty in *h* is just the relative uncertainty in vapour pressure (equation (2)) multiplied by the relative humidity; i.e.,

$$u\left(h\right) \simeq -0.3 \cdot t_{\mathrm{m}} \cdot h \quad \%\mathrm{rh}. \qquad (6)$$

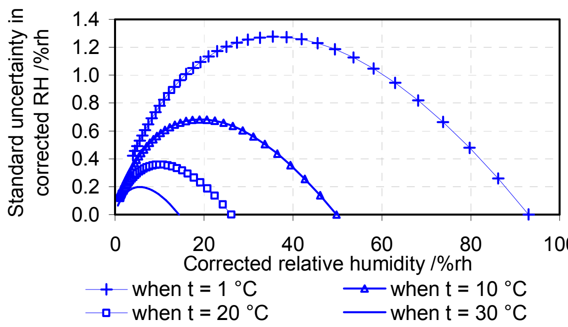

Figure 10. Standard uncertainty in corrected relative humidity calculated using the corrected dew point or frost point when the chilled-mirror condensate phase is unknown, for a range of dry-bulb temperatures.

The uncertainty in the corrected value of relative humidity calculated using equation (6) is shown in Figure 10 for a range of dry-bulb temperatures.

If the chilled-mirror hygrometer displays relative humidity, its internal calculation of *h* will be made on the assumption of either dew or frost on the mirror, and the maximum potential error will be as in Figure 9. There is no simple equation analogous to equation (3) to correct the displayed relative humidity. It is better to disregard the displayed relative humidity altogether and to calculate the corrected relative humidity from the dew point as described above; i.e., in the same way as must be done in the absence of a relative humidity display.

Conclusion

When the condensate phase on a chilled-mirror is unknown, significant error can arise in the determination of vapour pressure, dew and frost points, and relative humidity. Potential error and measurement uncertainty can be reduced by applying the corrections with uncertainties as outlined.

References

[1] *A Guide to the Measurement of Humidity*, Institute of Measurement and Control, London, 1996.

[2] D Sonntag, "Advancements in the field of hygrometry", *Zeitschrift fur Meteorologie*, **3**(2), 51–66, 1994.

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